OKLAHOMA STATE UNIVERSITY

SCHOOL OF ELECTRICAL AND COMPUTER ENGINEERING

ECEN 5713 System Theory
Fall 1997
Midterm Exam #2

Name : ______________________________

Student ID: ________________________________

E-Mail Address:_______________________________________

Problem 1:
Let

$$S = \left\{ x | x = a\begin{bmatrix}1\\2\\0\end{bmatrix} + b\begin{bmatrix}0\\1\\2\end{bmatrix}, a, b \in \Re \right\},$$

find the orthogonal complement space of S, $S^{\perp}(\subset \Re^3)$, and determine an orthonormal basis and dimension for $S^{\perp}$. For $x = \begin{bmatrix}1 & 2 & 3\end{bmatrix}^T (\in \Re^3)$. And find its direct sum representation of $x = x_1 \oplus x_2$, such that $x_1 \in S, x_2 \in S^{\perp}$.

Problem 2:

Let $V = F^3$, and let F be the field of rational polynomials. Determine the representation of $v = \begin{bmatrix} s+2 & \frac{1}{s} & -2 \end{bmatrix}^T$ in (V, F) with respect to the basis $\{v^1, v^2, v^3\}$, where $v^1 = \begin{bmatrix} 1 & -1 & 0 \end{bmatrix}^T$, $v^2 = \begin{bmatrix} 1 & 0 & -1 \end{bmatrix}^T$, $v^3 = \begin{bmatrix} 0 & 1 & 0 \end{bmatrix}^T$.

<u>Problem 3</u>:

$$A = \begin{bmatrix} 1 & 2 & 3 & 4 & 5 \\ 2 & 3 & 4 & 1 & 2 \\ 3 & 4 & 5 & 0 & 0 \end{bmatrix}$$

What are the rank and nullity of the above linear operator, A ? And find the bases of the range spaces and the null spaces of the operator, A ?

<u>Problem 4</u>:

Show that the determinant of the $m \times m$ matrix

$$\begin{bmatrix} s^{k_m} & -1 & 0 & \cdots & 0 & 0 \\ 0 & s^{k_{m-1}} & -1 & \cdots & 0 & 0 \\ 0 & 0 & s^{k_{m-2}} & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & \cdots & s^{k_2} & -1 \\ \beta_m(s) & \beta_{m-1}(s) & \beta_{m-2}(s) & \cdots & \beta_2(s) & s^{k_1} + \beta_1(s) \end{bmatrix}$$

is equal to

$$s^n + \beta_1(s)s^{n-k_1} + \beta_2(s)s^{n-k_1-k_2} + \cdots + \beta_m(s)$$

where $n = k_1 + k_2 + \cdots + k_m$ and $\beta_i(s)$ are arbitrary polynomials.

(<u>hint</u>: proof by induction)